CHAPTER 12

Statistics and Probability

Knowledge and Skills

- Evaluate predictions and conclusions based on statistical data. (Reinforcement of TEKS 8.13)
- Apply concepts of theoretical and experimental probability to make predictions. (Reinforcement of TEKS 8.11)

Key Vocabulary

combination (p. 649)
compound event (p. 655)
permutation (p. 647)
sample (p. 634)

Real-World Link

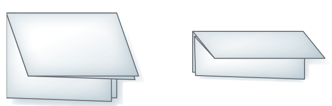
U.S. Senate The United States Senate forms committees to focus on different issues. These committees are made up of senators from various states and political parties. You can use probability to find how many ways these committees can be formed.

FOLDABLES Study Organizer

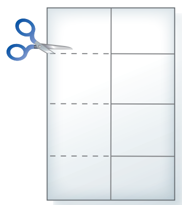
Statsitics and Probability Make this Foldable to help you organize what you learn about statistics and probability. Begin with a sheet of $8\frac{1}{2}$" × 11" paper.

1. **Fold** in half lengthwise.

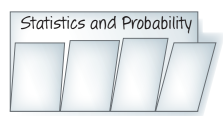
2. **Fold** the top to the bottom twice.



3. **Open.** Cut along the second fold to make four tabs.



4. **Label** as shown.



GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at tx.algebra1.com.

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICKQuiz

Determine the probability of each event if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, 3 blue cubes, and 1 green cube. (For use in Lesson 12-2 and 12-5.)

1. $P(\text{red})$ $\frac{3}{7}$
2. $P(\text{blue})$ $\frac{3}{14}$
3. $P(\text{not red})$ $\frac{4}{7}$
4. **GAMES** Paul is going to roll a game cube with 3 sides painted red, two painted blue, and 1 painted green. What is the probability that a red side will land face up? $\frac{1}{2}$

Find each product. (Used in Lesson 12-2.)

5. $\frac{4}{5} \cdot \frac{3}{4}$ $\frac{3}{5}$
6. $\frac{5}{12} \cdot \frac{6}{11}$ $\frac{5}{22}$
7. $\frac{7}{20} \cdot \frac{4}{19}$ $\frac{7}{95}$
8. $\frac{4}{32} \cdot \frac{7}{32}$ $\frac{7}{256}$
9. $\frac{13}{52} \cdot \frac{4}{52}$ $\frac{1}{52}$
10. $\frac{56}{100} \cdot \frac{24}{100}$ $\frac{84}{625}$

Write each fraction as a percent. Round to the nearest tenth. (Used in Lesson 12-5)

11. $\frac{7}{8}$ 87.5%
12. $\frac{33}{80}$ 41.3%
13. $\frac{107}{125}$ 85.6%
14. $\frac{625}{1024}$ 61%
15. **CONCERTS** At a local concert, 585 of 2000 people were under the age of 18. What percentage of the audience were under 18? Round to the nearest tenth. 29.3%

QUICKReview

EXAMPLE 1

Determine the probability of selecting a green cube if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, and 1 green cube.

There is 1 green cube and a total of 11 cubes in the bag.

$$\frac{1}{11} = \frac{\text{number of green cubes}}{\text{total number of cubes}}$$

The probability of selecting a green cube is $\frac{1}{11}$.

EXAMPLE 2

Find $\frac{5}{4} \cdot \frac{2}{3}$.

$\frac{5}{4} \cdot \frac{2}{3} = \frac{5 \cdot 2}{4 \cdot 3}$ Multiply both the numerators and the denominators.

$= \frac{10}{12}$ or $\frac{5}{6}$ Simplify.

EXAMPLE 3

Write the fraction $\frac{14}{17}$ as a decimal. Round to the nearest tenth.

$\frac{14}{17} = 0.823$ Simplify and round.

0.823×100 Multiply the decimal by 100.

$= 82.3$ Simplify.

$\frac{14}{17}$ written as a percent is 82.3%.

For Exercises 20–22, roll two dice 50 times and record the sums.

20. See students' work.

20. Based on your results, what is the probability that the sum is 8?

21. Based on your results, what is the probability that the sum is 7, or the sum is greater than 5? See students' work.

22. If you roll the dice 25 more times, which sum would you expect to see about 10% of the time? 5 or 9

RESTAURANTS **For Exercises 23–25, use the following information.**
A family restaurant gives away a free toy with each child's meal. There are eight different toys that are randomly given. There is an equally likely chance of getting each toy each time. 23. Sample answer: 4 coins

23. What objects could be used to perform a simulation of this situation?

24. Conduct a simulation until you have one of each toy. Record your results.

25. Based on your results, how many meals must be purchased so that you get all 8 toys?

24–25. See students' work.

Real-World Link
Labrador retrievers are the most popular breed of dog in the United States.
Source: American Kennel Club

ANIMALS **For Exercises 26–29, use the following information.**
Refer to Example 4 on page 671. Suppose Ali's dog has a litter of 5 puppies.

26. List the possible outcomes of the genders of the puppies. See margin.

27. Perform a simulation and list your results in a table. See students' work.

28. Based on your results, what is the probability that there are 3 females and two males in the litter? See students' work.

29. What is the experimental probability that the litter has at least three males? See students' work.

ENTERTAINMENT **For Exercises 30–32, use the following information.**
A CD changer contains 5 CDs with 14 songs each. When "Random" is selected, each CD is equally likely to be chosen as each song. 30–32. See students' work.

30. Use a graphing calculator to perform a simulation of randomly playing 20 songs from the 5 CDs. Record your answer.
KEYSTROKES: MATH ◀ 5 1 , 70 , 20) ENTER

31. Do the experimental probabilities for your simulation support the statement that each CD is equally likely to be chosen? Explain.

32. Based on your results, what is the probability that the first three songs played are on the third disc?

EXTRA PRACTICE
See page 733, 745.

Math Online
Self-Check Quiz at tx.algebra1.com

H.O.T. Problems

33. Sample answer: a survey of 100 people voting in a two-person election where 50% of the people favor each candidate; 100 coin tosses

34. No; there were 181 heads out of the 300 tosses. The experimental probability of heads is about 60%.

33. **OPEN ENDED** Describe a real-life situation that could be represented by a simulation. What objects would you use for this experiment?

34. **CHALLENGE** The captain of a football team believes that the coin the referee uses for the opening coin toss gives an advantage to one team. The referee has players toss the coin 50 times each and record their results. Based on the results, do you think the coin is fair? Explain your reasoning.

Player	1	2	3	4	5	6
Heads	38	31	29	27	26	30
Tails	12	19	21	23	24	20

35. *Writing in Math* Refer to the information on page 669 to explain how simulations can be used in health care. Include an explanation of experimental probability and why more trials are better than fewer trials when considering experimental probability. See margin.

TEST PRACTICE
TAKS 9, 10

36. Ramón tossed two coins and rolled a die. What is the probability that he tossed two tails and rolled a 3? D

A $\frac{1}{4}$ C $\frac{5}{12}$

B $\frac{1}{6}$ D $\frac{1}{24}$

37. **GRADE 8 REVIEW** Blair Kastanza runs a day care and every 4 years the number of children that he cares for triples. If the pattern continues, and he originally started with 5 children, approximately how many children will he be caring for in 20 years? J

F 23 H 125

G 67 J 1215

Spiral Review

For Exercises 38–40, use the probability distribution for the random variable X, the number of computers per household. (Lesson 12-5)

Computers per Household	
X = Number of Computers	$P(X)$
0	0.579
1	0.276
2	0.107
3+	0.038

Source: U.S. Dept. of Commerce

38. Show that the probability distribution is valid. $0.579 + 0.276 + 0.107 + 0.038 = 1$

39. If a household is chosen at random, what is the probability that it has at least 2 computers? 0.145

40. Determine the probability of randomly selecting a household with no more than one computer. 0.855

For Exercises 41–43, use the following information.
A jar contains 18 nickels, 25 dimes, and 12 quarters. Three coins are randomly selected one at a time. Find each probability. (Lesson 12-4)

41. picking three dimes, replacing each after it is drawn $\frac{125}{1331}$

42. a nickel, then a quarter, then a dime without replacing the coins $\frac{20}{583}$

43. 2 dimes and a quarter, without replacing the coins, if order does not matter $\frac{80}{583}$

Determine whether the following side measures would form a right triangle. (Lesson 10-4)

44. 5, 7, 9 no

45. $3\sqrt{34}$, 9, 15 yes

46. 36, 86.4, 93.6 yes

Cross-Curricular Project

Algebra and Physical Science

Building the Best Roller Coaster It is time to complete your project. Use the information and data you have gathered about the building and financing of a roller coaster to prepare a portfolio or Web page. Be sure to include graphs, tables, and/or calculations in the presentation.

Math Online **Cross-Curricular Project at** tx.algebra1.com

Texas Test Practice

Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The table shows the results of a survey given to 600 customers at a music store.

Favorite Music	Percent
Jazz	12
Pop	58
Classical	14
Other	16

Based on these data, which of the following statements is true? **B**

A More than half of the customers' favorite music is classical or jazz.

B More customers' favorite is pop music than all other types of music.

C More customers' favorite music is something other than jazz, pop, or classical.

D The number of customers whose favorite music is pop is more than five times the number of customers whose favorite music is jazz.

2. Which equation describes a line that has a y-intercept of -3 and a slope of 6? **J**

F $y = -3x + 6$

G $y = (-3 + x)6$

H $y = (-3x + 1)6$

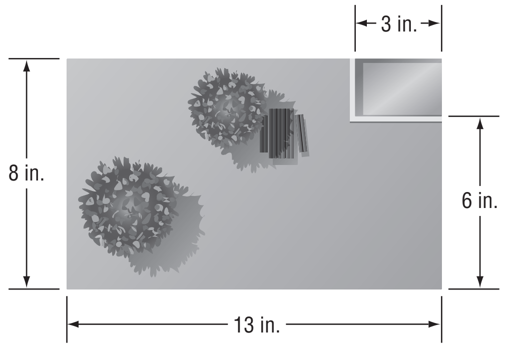
J $y = 6x - 3$

TEST-TAKING TIP

Question 2 Know the slope-intercept form of linear equations, $y = mx + b$, and understand the definition of slope.

3. GRIDDABLE Hailey is driving her car at a rate of 65 miles per hour. What is her rate in miles per second? Round to the nearest hundredth. **0.02**

4. Sally is ordering a cover for her swimming pool pictured below. The cover costs $5.00 per square foot.



What information must be provided in order to find the total cost of the cover? **C**

A The width of the swimming pool.

B The thickness of the cover.

C The scale of inches to feet on the drawing.

D The amount that Sally has budgeted for the cover.

5. At Marvin's Pizza Place, 30% of the customers order pepperoni pizza. Also, 65% of the customers order a cola to drink. What is the probability that a customer selected at random orders a pepperoni pizza and a cola? **J**

F $\frac{95}{100}$

G $\frac{1}{3}$

H $\frac{35}{100}$

J $\frac{39}{200}$

6. When graphed, which function would appear to be shifted 3 units down from the graph of $f(x) = x^2 + 2$? **D**

A $f(x) = x^2 + 5$

B $f(x) = x^2$

C $f(x) = x^2 - 3$

D $f(x) = x^2 - 1$

7. GRIDDABLE Laura's Pizza Shop has your choice of 5 meats, 3 cheeses, and 4 vegetables. How many different combinations are there if you choose 1 meat, 1 cheese, and 1 vegetable? **60**

Math Online Texas Test Practice at tx.algebra1.com

Get Ready for the Texas Test
For test-taking strategies and more practice, see pages TX1–TX35.

8. Carlos rolled a 6 sided die 60 times. The results of his rolls are shown in the table.

Number	Frequency
1	12
2	9
3	14
4	10
5	7
6	8

What is the difference between the theoretical probability and the experimental probability for rolling a 5? **F**

F 5%

G 12%

H 17%

J 30%

9. Miguel ro\lled a six-sided die 60 times. The results are shown in the table below.

Side	Number of Times Landed
1	12
2	10
3	8
4	15
5	7
6	8

Which number has the same experimental probability as theoretical probability? **B**

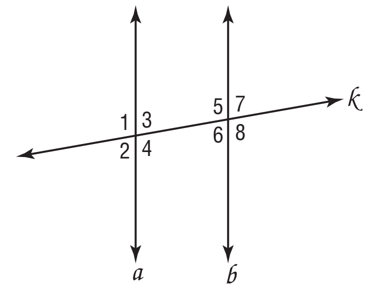
A 1

B 2

C 3

D 4

10. Lines a and b are parallel. Both lines are cut by transversal k. **J**



Which statement is not a valid conclusion?

F $\angle 1 \cong \angle 8$

G $\angle 1 \cong \angle 5$

H $\angle 3 \cong \angle 6$

J $\angle 4 \cong \angle 7$

11. Maryn is filling up the water can pictured below. How many times more water could she fit in the can if she doubled the radius? **C**

A 2 times

B 3 times

C 4 times

D 8 times

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. At WackyWorld Pizza, the Random Special is a random selection of two different toppings on a large cheese pizza. The available toppings are pepperoni, sausage, onion, mushrooms, and green peppers.

a. How many different Random Specials are possible? Show how you found your answer. **See margin.**

b. If you order the Random Special, what is the probability that it will have onions? **See margin.**

c. If you order the Random Special, what is the probability that it will have neither onion nor green peppers? **See margin.**

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	12-6	4-3	11-4	2-4	12-4	9-1	12-4	12-6	12-6	TX33	TX27	12-2 and 12-3
For Help with Test Objective	9	3	9	10	9	5	10	9	9	10	8	9

12-2 Permutations and Combinations

Main Ideas

- Solve problems involving permutations.
- Solve problems involving combinations.

Reinforcement of TEKS 7.10
The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real-life events.
(A) Construct sample spaces for simple or composite experiments.

New Vocabulary

permutation
linear permutation
combination

GET READY for the Lesson

When the manager of a softball team fills out her team's lineup card before the game, the order in which she fills in the names is important because it determines the order in which the players will bat.

Suppose she has 7 possible players in mind for the top 4 spots in the lineup. You know from the Fundamental Counting Principle that there are $7 \cdot 6 \cdot 5 \cdot 4$ or 840 ways that she could assign players to the top 4 spots.

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**. In a permutation, the *order* of the objects is very important. The arrangement of objects or people in a line is called a **linear permutation**.

Notice that $7 \cdot 6 \cdot 5 \cdot 4$ is the product of the first 4 factors of 7!. You can rewrite this product in terms of 7!.

$$7 \cdot 6 \cdot 5 \cdot 4 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \qquad \text{Multiply by } \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or 1.}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or } \frac{7!}{3!} \qquad 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ and } 3! = 3 \cdot 2 \cdot 1$$

Notice that 3! is the same as $(7 - 4)!$.

The number of ways to arrange 7 people or objects taken 4 at a time is written $P(7, 4)$. The expression for the softball lineup above is a case of the following formula.

Reading Math

Permutations The expression $P(n, r)$ reads *the number of permutations of n objects taken r at a time.* It is sometimes written as ${}_nP_r$.

KEY CONCEPT *Permutations*

The number of permutations of n distinct objects taken r at a time is given by

$$P(n, r) = \frac{n!}{(n - r)!}.$$

EXAMPLE Permutation

1 FIGURE SKATING There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?

Since each winner will receive a different medal, order is important. You must find the number of permutations of 10 things taken 3 at a time.

Study Tip

Alternate Method

Notice that in Example 1, all of the factors of $(n - r)!$ are also factors of $n!$. You can also evaluate the expression in the following way.

$$\frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 \text{ or } 720$$

$$P(n, r) = \frac{n!}{(n - r)!} \qquad \text{Permutation formula}$$

$$P(10, 3) = \frac{10!}{(10 - 3)!} \qquad n = 10, r = 3$$

$$= \frac{10!}{7!} \qquad \text{Simplify.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7}^{1} \cdot \cancel{6}^{1} \cdot \cancel{5}^{1} \cdot \cancel{4}^{1} \cdot \cancel{3}^{1} \cdot \cancel{2}^{1} \cdot \cancel{1}^{1}}{\cancel{7}_{1} \cdot \cancel{6}_{1} \cdot \cancel{5}_{1} \cdot \cancel{4}_{1} \cdot \cancel{3}_{1} \cdot \cancel{2}_{1} \cdot \cancel{1}_{1}} \text{ or } 720 \qquad \text{Divide by common factors.}$$

The gold, silver, and bronze medals can be awarded in 720 ways.

CHECK Your Progress

1. A newspaper has nine reporters available to cover four different stories. How many ways can the reporters be assigned to cover the stories? **3024**

Suppose you want to rearrange the letters of the word *geometry* to see if you can make a different word. If the two *e*s were not identical, the eight letters in the word could be arranged in $P(8, 8)$ ways. To account for the identical *e*s, divide $P(8, 8)$ by the number of arrangements of *e*. The two *e*s can be arranged in $P(2, 2)$ ways.

$$\frac{P(8, 8)}{P(2, 2)} = \frac{8!}{2!} \qquad \text{Divide.}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} \text{ or } 20{,}160 \qquad \text{Simplify.}$$

Thus, there are 20,160 ways to arrange the letters in *geometry*.

When some letters or objects are alike, use the rule below to find the number of permutations.

KEY CONCEPT *Permutations with Repetitions*

The number of permutations of n objects of which p are alike and q are alike is

$$\frac{n!}{p!q!}.$$

This rule can be extended to any number of objects that are repeated.

EXAMPLE Permutation with Repetition

2 How many different ways can the letters of the word *MISSISSIPPI* be arranged?

The letter *I* occurs 4 times, *S* occurs 4 times, and *P* occurs twice.

You need to find the number of permutations of 11 letters of which 4 of one letter, 4 of another letter, and 2 of another letter are the same.

$$\frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!4!2!} \text{ or } 34{,}650$$

There are 34,650 ways to arrange the letters.

CHECK Your Progress

2. How many different ways can the letters of the word *DECIDED* be arranged? **420**

Measures of Variation **Measures of variation** or **dispersion** measure how spread out or scattered a set of data is. The simplest measure of variation to calculate is the *range*, the difference between the greatest and the least values in a set of data. Variance and standard deviation are measures of variation that indicate how much the data values differ from the mean.

Reading Math

Symbols The symbol σ is the lower case Greek letter *sigma*. $\bar{x}$ is read *x bar*.

To find the **variance** σ^2 of a set of data, follow these steps.

1. Find the mean, $\bar{x}$.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.

The **standard deviation** σ is the square root of the variance.

KEY CONCEPT *Standard Deviation*

If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

EXAMPLE Standard Deviation

2 **STATES** **The table shows the populations in millions of 11 eastern states as of the 2000 Census. Find the variance and standard deviation of the data to the nearest tenth.**

State	Population	State	Population	State	Population
NY	19.0	MD	5.3	RI	1.0
PA	12.3	CT	3.4	DE	0.8
NJ	8.4	ME	1.3	VT	0.6
MA	6.3	NH	1.2	–	–

Source: U.S. Census Bureau

Step 1 Find the mean. Add the data and divide by the number of items.

$$\bar{x} = \frac{19.0 + 12.3 + 8.4 + 6.3 + 5.3 + 3.4 + 1.3 + 1.2 + 1.0 + 0.8 + 0.6}{11}$$

$$\approx 5.4\overline{18}$$ The mean is about 5.4 million people.

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$ Variance formula

$$\approx \frac{(19.0 - 5.4)^2 + (12.3 - 5.4)^2 + \cdots + (8.0 - 5.4)^2 + (0.6 - 5.4)^2}{11}$$

$$\approx \frac{344.4}{11}$$ Simplify.

$$\approx 31.3\overline{09}$$ The variance is about 31.3.

Step 3 Find the standard deviation.

$\sigma^2 \approx 31.3$ Take the square root of each side.

$\sigma \approx 5.594640292$ The standard deviation is about 5.6 million people.

CHECK Your Progress

2. The leading number of home runs in Major League Baseball for the 1994-2004 seasons were 43, 50, 52, 56, 70, 65, 50, 73, 57, 47, and 48. Find the variance and standard deviation of the data to the nearest tenth. **87.9, 9.4**

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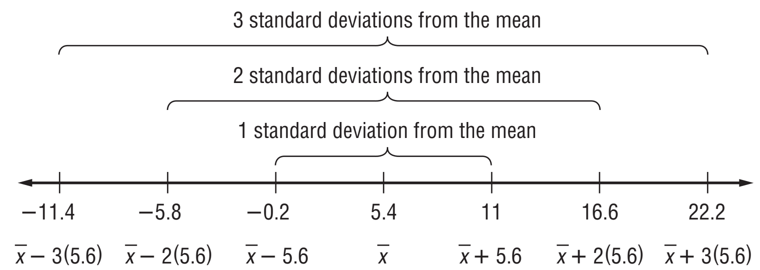
Most of the members of a set of data are within 1 standard deviation of the mean. The data in Example 2 can be broken down as shown below.



Looking at the original data, you can see that most of the states' populations were between 2.4 million and 20.2 million. That is, the majority of members of the data set were within 1 standard deviation of the mean.

You can use a TI-83/84 Plus graphing calculator to find statistics for the data in Example 2.

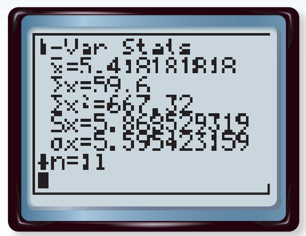
GRAPHING CALCULATOR LAB

One-Variable Statistics

The TI-83/84 Plus can compute a set of one-variable statistics from a list of data. These statistics include the mean, variance, and standard deviation. Enter the data into L1.

KEYSTROKES: STAT ENTER 19.0 ENTER 12.3 ENTER . . .

Then use STAT ▶ 1 ENTER to show the statistics. The mean $\bar{x}$ is about 5.4, the sum of the values $\sum x$ is 59.6, the standard deviation σx is about 5.6, and there are $n = 11$ data items. If you scroll down, you will see the least value (minX = .6), the three quartiles (1, 3.4, and 8.4), and the greatest value (maxX = 19).



THINK AND DISCUSS

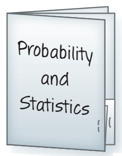
1. Find the variance of the data set. **about 31.36**
2. Enter the data set in list L1 but without the outlier 19.0. What are the new mean, median, and standard deviation? **4.06, 2.35, about 3.8**
3. Did the mean or median change less when the outlier was deleted? **median**

Chapter 12 Study Guide and Review

STUDY TO GO Download Vocabulary Review from tx.algebra2.com

FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

The Counting Principle, Permutations, and Combinations (Lessons 12-1 and 12-2)

- Fundamental Counting Principle: If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.
- Permutation: order of objects is important.
- Combination: order of objects is not important.

Probability (Lessons 12-3 and 12-4)

- Two independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Two dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$
- Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
- Inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Statistical Measures (Lesson 12-5)

- To represent a set of data, use the mean if the data are spread out, the median when the data has outliers, or the mode when the data are tightly clustered around one or two values.
- Standard deviation for n values: $\bar{x}$ is the mean,

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

The Normal Distribution (Lesson 12-6)

- The graph is maximized at the mean and the data are symmetric about the mean.

Binomial Experiments, Sampling, and Error (Lessons 12-7 and 12-8)

- A binomial experiment exists if and only if there are exactly two possible outcomes, a fixed number of independent trials, and the possibilities for each trial are the same.

Key Vocabulary

binomial experiment (p. 730)
combination (p. 692)
compound event (p. 710)
dependent events (p. 686)
event (p. 684)
inclusive events (p. 712)
independent events (p. 684)
measure of variation (p. 718)
mutually exclusive events (p. 710)
normal distribution (p. 724)
outcome (p. 684)
permutation (p. 690)
probability (p. 697)
probability distribution (p. 699)
random (p. 697)
random variable (p. 699)
relative-frequency histogram (p. 699)
sample space (p. 684)
simple event (p. 710)
standard deviation (p. 718)
unbiased sample (p. 735)
uniform distribution (p. 699)
univariate data (p. 717)
variance (p. 718)

Vocabulary Check

Choose the term that best matches each statement or phrase. Choose from the list above.

1. the ratio of the number of ways an event can succeed to the number of possible outcomes **probability**
2. an arrangement of objects in which order does not matter **combination**
3. two or more events in which the outcome of one event affects the outcome of another event **dependent events**
4. a sample in which every member of the population has an equal chance to be selected **unbiased sample**
5. two events in which the outcome can never be the same **mutually exclusive events**
6. an arrangement of objects in which order matters **permutation**
7. the set of all possible outcomes **sample space**
8. an event that consists of two or more simple events **compound event**

Math Online Vocabulary Review at tx.algebra2.com

Lesson-by-Lesson Review

12–1 The Counting Principle (pp. 684–689)

9. **PASSWORDS** The letters a, c, e, g, i, and k are used to form 6-letter passwords. How many passwords can be formed if the letters can be used more than once in any given password? **46,656 passwords**

Example 1 How many different license plates are possible with two letters followed by three digits?

There are 26 possibilities for each letter. There are 10 possibilities, the digits 0–9, for each number. Thus, the number of possible license plates is as follows.

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$ or 676,000

12–2 Permutations and Combinations (pp. 690–695)

10. A committee of 3 is selected from Jillian, Miles, Mark, and Nikia. How many committees contain 2 boys and 1 girl? **2**
11. Five cards are drawn from a standard deck of cards. How many different hands consist of four queens and one king? **4**
12. A box of pencils contains 4 red, 2 white, and 3 blue pencils. How many different ways can 2 red, 1 white, and 1 blue pencil be selected? **36**

Example 2 A basket contains 3 apples, 6 oranges, 7 pears, and 9 peaches. How many ways can 1 apple, 2 oranges, 6 pears, and 2 peaches be selected?

This involves the product of four combinations, one for each type of fruit.

$C(3, 1) \cdot C(6, 2) \cdot C(7, 6) \cdot C(9, 2)$

$= \frac{3!}{(3-1)!1!} \cdot \frac{6!}{(6-2)!2!} \cdot \frac{7!}{(7-6)!6!} \cdot \frac{9!}{(9-2)!2!}$

$= 3 \cdot 15 \cdot 7 \cdot 36$ or 11,340 ways

12–3 Probability (pp. 697–702)

13. A bag contains 4 blue marbles and 3 green marbles. One marble is drawn from the bag at random. What is the probability that the marble drawn is blue? $\frac{4}{7}$
14. **COINS** The table shows the distribution of the number of heads occurring when four coins are tossed. Find $P(H = 3)$. $\frac{1}{4}$

H = Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Example 3 A bag of golf tees contains 23 red, 19 blue, 16 yellow, 21 green, 11 orange, 19 white, and 17 black tees. What is the probability that if you choose a tee from the bag at random, you will choose a green tee?

There are 21 ways to choose a green tee and 23 + 19 + 16 + 11 + 19 + 17 or 105 ways not to choose a green tee. So, s is 21 and f is 105.

$P(\text{green tee}) = \frac{s}{s+f}$

$= \frac{21}{21+105}$ or $\frac{1}{6}$